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Astroparticle Physics (Physics 711)
Exercise 4

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Problem 1: Friedmann Cosmology (5P)

Calculate the difference between the actual density and the critical density $\Delta\rho/\rho = (\rho - \rho_c)/\rho$ as a function of the density ρ and the scale factor a for arbitrary curvature. Now consider the energy scale (T=10^{14} GeV) where grand unification may have occured. Determine $\Delta\rho/\rho$ at this energy scale, remember that $\rho = \rho_{\mathrm{M}} a^{-3} + \rho_\Lambda + \rho_{\mathrm{R}} a^{-4}$ (with $\Omega_{\mathrm{M}} = 0.26$, $\Omega_\Lambda = 0.74$ and $\Omega_{\mathrm{R}} = 4.8 \cdot 10^{-8}$) and use the approxmiation $a^{-1} \sim T$ ($T_{\mathrm{o}} = 2.7$ K $\sim 10^{-4}$ eV). Compare the magnitude of Ω_{k} of this era to today's value ($\Omega_{\mathrm{k}} = 0.01 \pm 0.01$) and discuss your results.

Problem 2: Quintessence (4P)

A hypothetical description of dark energy is quintessence. According to this, dark energy is represented by a time dependent scalar field.
Assume dark energy is described by the scalar field $\Phi = \Phi_0 \ln t$ with potential $V(\Phi) = e^{-\Phi}$ and energy density $\rho_{de} = V(\Phi) + \frac{1}{2}(d\Phi/dt)^2$. The equation of motion satisfying the Einstein equations reads

$$\frac{d^2\Phi}{dt^2} + 3H\frac{d\Phi}{dt} + \frac{dV}{d\Phi} = 0. \tag{1}$$

Show that $\rho_{de} \propto \frac{1}{t^2}$ using the ansatz $a = t^\beta$.