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Astroparticle Physics (Physics 711)

Exercise 2

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Problem 1: Critical Density and Age of the Universe(5P)

a) For any value of $H_0 = \dot{a}(t_0)/a(t_0)$ we can define a critical density $\rho_0^{\text{crit}} = (3H_0^2)/(8\pi G)$. Calculate the value of the critical density and compare it to the density of earth's atmosphere, the density of the sun and the density of the best laboratory vacuum. What is the corresponding energy scale of the critical density? Which Standard Model particles best match the scale?

b) Calculate (analytically) the age of a flat Universe if radiation is neglected and it is presently made up of matter with Ω_M=0.3 and vacuum energy with Ω_Λ=0.7. Use the substitution $x = (1+z)^3\,\Omega_m/(1-\Omega_m)$.

Problem 2: Friedmann Cosmology (5P)

Assuming an expanding and flat (K=0) FLRW universe, the first Friedmann equation reads

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G\rho}{3}. \tag{1}$$

a) Compute the fraction of elapsed times t_{mat} and t_{rad} to reach an energy density ρ in a matter and a radiation dominated universe, respectively (hint: use Eqn.(1) to infer $R = R(t)$ and $\rho \propto R^n$ with n according to radiation or matter dominance to arrive at $\rho = \rho(t)$).

b) Show that for the density ρ_{ff} for a free-falling collapsing spherical gas cloud from rest at initial radius r_0 to a radius r after t_{rad}, $\rho_{\text{ff}} > \rho_{\text{rad}}$ holds ($\rho_{\text{rad}} = \rho(t_{\text{rad}})$ as computed above). (hint: equate the loss in gravitational energy with the gain in kinetic energy of a small test mass m in the outermost shell of the cloud. Use the substitution $r = r_0 \sin^2\theta$ for the integral $t = \int \frac{dr}{dr/dt}$.)